# Computer Science Education in the Montessori Classroom 

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## Preliminaries

- Slides will be posted at:
- cse4k12.blogspot.com
- Email contact:
- garykac @ either "gmail" or "google" .com

This info will be repeated at end

## Background

- Desire to teach computer programming
- Taught class:
- 6th-8th graders
- ~40 once-a-week 1-hour classes
- Student project: create a GBA/NDS game
- First few months of class spent teaching basics
- These could/should have been taught earlier


## Student projects



## Why teach computer science?

- Well, why do we:
- Teach the Bohr model of the atom?
- Teach photosynthesis?
- ... other sciences?
- Answer:
- So students can understand the world around them


## Understanding your surroundings

- How does:
- A toilet work?
- A phone work?
- A car (internal combustion engine) work?
- We can explain how these work in general, accessible terms.
- Concrete physical explanation


## How do computers work?

- Many explanations are something like:
- Mumbling something about 0's and 1's
- Memory, hard drives, CPU, ALU
- These aren't wrong, but they aren't concrete
- Students don't walk away with understanding


## It's all magic

- "Any sufficiently advanced technology is indistinguishable from magic."
- Arthur C. Clark, 1973
- Kids are growing up in this magical world
- How much of the world will they understand in 20, 30 years?
- Compare this with 100 years ago


## But we teach computers, don't we?

- Current state of K-8 computer "science":
- Typing skills
- How to use productivity applications
- Word processing, spreadsheets, ...
- These are necessary, but not sufficient
- We don't teach students how to use a calculator
- We teach them math


## But programming is hard/abstract

- True, so teach programming in 6th-8th
- Requires abstract thinking
- But teach pre-programming before 6th grade
- Focus on fundamental principles:
- Binary
- Logic
- Transistors
- No need for students to use computers for this


## Our goals

- Convince you that teaching pre-programming:
- Is important
- Is possible
- Is easy
- Well, as easy as any other topic you teach


## Pre-programming skills

- Binary / Hexadecimal
- Boolean Logic
- Transistors
- How Computers Add
- Not a skill per se
- But useful to tie everything together


## Binary / Hexadecimal (3rd-4th grade)

## Number systems

－How many ways can you represent a number？
－Tally marks：｜｜｜｜｜｜
－Roman numerals：LXVII
－Writing systems：ーニ三四 ．．．IYケ乏．．．
－Activity：
－Grab random number of counters and write the \＃as many different ways as possible
－Why：To demonstrate that decimal is not the only way to count things

## Zero

- The concept of zero is a crucial part of our number system
- Before zero, how do you distinguish between:
- There were no items
- Oops, I forgot to fill in this value
- Curriculum link:
- Compare early societies that invented zero vs. those that didn't


## Positional notation

- Decimal is a base-10 positional notation
- There are 10 digits: 0123456789
- Value of each digit depends on its position:

$$
\begin{array}{lr}
\cdot \\
-\ldots, 1000, \quad 100, \quad 10,1 \\
\cdot \ldots, 10 \times 10 \times 10,10 \times 10,10,1
\end{array}
$$

- Compare 324 and 243
- Both use same digits
- Value of each digit is different


## Why base-10?

- What about other bases?
- Some cultures use/used base-5, -12 or -60
- Activity:
- Where do we see remnants of other bases in modern society?
- Clocks, Angles, Lat/Long, Eggs
- Why: Point out that the choice of 10 was somewhat arbitrary


## Octal (base-8)

- 8 digits: 01234567
- Positions: ..., 8x8x8, 8x8, 8, 1
- Why octal?
- Binary can be confusing at first.
- Octal is closer to decimal - less confusing
- Activity:
- Get random number of counters. Group by 10 to get decimal. Using same counters, group by 8 to get octal. Repeat.


## Counting in octal

- $0,1,2,3,4,5,6,7,10,11,12,13,14,15,16$, $17,20,21, \ldots, 77,100, \ldots$
- 10 comes after 7
- 20 comes after 17
- 100 comes after 77
- Compare with decimal
- 8, 9 are never used


## How many is 12?

- Draw 12 circles
- Ambiguous - base-10 or base-8?
- How many circles?
- 10 (in octal)
- 8 (in decimal)



## Ambiguity

- " 12 " means different things in octal/decimal
- How do we know which one to use?
- Compare:
- How do you pronounce "wind"?
- Is "rose" a noun or a verb?
- "I'll meet you at 8 ". Is that AM or PM?
- Context resolves the ambiguity.


## Binary (base-2)

- Same as decimal, octal, except:
- 2 digits: 0,1
- Positions: ..., $2 \times 2 \times 2 \times 2,2 \times 2 \times 2,2 \times 2,2,1$


## Counting in binary

- 0,1
- Oh no, we ran out of digits already
- 10,11
- We ran out again
- $100,101,110,111$
- $1000,1001,1010,1011,1100,1101,1110$, 1111
- This is why we start with octal


## Binary activity

- Grab a random number of counters (20-30)
- Group by 2's
- Combine these into groups of 4's
- Repeat into groups of 8's, 16's, 32's
- Place group next to binary position card
- Binary \# is created by:
- Writing a " 1 " where you have a group
- Writing a " 0 " where you don't


## Binary activity (example)

- Example, take 11 tokens: 00000000000
- Group by 2: (oo)(oo)(oo)(oo)(oo)o
- Group by 4: (oooo)(oooo)(oo)o
- Group by 8: (oooooooo)(oo)o
- We have: 1 eight, 0 four, 1 two, 1 one
- Binary number: 1011


## Problems with binary

- Decimal is easier to work with
- 265 versus 100001001
- What is $11010101101+10110100101$ ?
- Non-trivial conversion between base-10


## Converting binary/decimal

- Remember, decimal 243 is:

$$
-2 \times 100+4 \times 10+3 \times 1
$$

- Binary 1011001 is:
$-1 \times 64+0 \times 32+1 \times 16+1 \times 8+0 \times 4+0 \times 2+1 \times 1$
- $=89$
- Ugh! That was work...


## Hexadecimal (base-16)

- 16 digits: 0123456789 abcdef
$-\mathrm{a}=10, \mathrm{~b}=11, \mathrm{c}=12, \mathrm{~d}=13, \mathrm{e}=14, \mathrm{f}=15$
- Single digit representation for each value
- Positions: ..., 16x16x16, 16x16, 16, 1
- Activity: (older students)
- Where have you seen hexadecimal?
- Specifying RGB colors: HTML, Photoshop, ...


## Counting in hexadecimal

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | $1 a$ | $1 b$ | 1c | 1d | 1e | 1f |
| 20 | 21 | 22 | $\ldots$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $\ldots$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e0 | e1 | e2 | e3 | e4 | e5 | e6 | e7 | e8 | e9 | ea | eb | ec | ed | ee | ef |
| f0 | f1 | f2 | f3 | f4 | f5 | f6 | f7 | f8 | f9 | fa | fb | fc | fd | fe | ff |
| 100 | 101 | 102 | $\ldots$ |  |  |  |  |  |  |  |  |  |  |  |  |

## Why hexadecimal?

- Compare positions for binary and hexadecimal
- They line up at $1,16,256, \ldots$
- This makes conversion easy

| binary | hexadecimal |
| :---: | :---: |
| 1 | 1 |
| 2 |  |
| 4 |  |
| 8 | 16 |
| 16 |  |
| 32 |  |
| 64 |  |
| 128 |  |
| 256 |  |
| 512 |  |
| 1024 |  |
| 2048 |  |

## Converting binary/hexadecimal

- Binary number: 1110100100100101
- Group by 4 digits: $(1110)(1001)(0010)(0101)$
- Starting from left side
- Convert each group independently:

| binary | hex |
| :---: | :---: |
| 0000 | 0 |
| 0001 | 1 |
| 0010 | 2 |
| 0011 | 3 |


| binary | hex |
| :---: | :---: |
| 0100 | 4 |
| 0101 | 5 |
| 0110 | 6 |
| 0111 | 7 |


| binary | hex |
| :---: | :---: |
| 1000 | 8 |
| 1001 | 9 |
| 1010 | a |
| 1011 | b |


| binary | hex |
| :---: | :---: |
| 1100 | c |
| 1101 | d |
| 1110 | e |
| 1111 | f |

- Thus, 1110100100100101 becomes E925


## Hexadecimal activity

- Activity: Take a set of binary numbers
- Convert them into either decimal or hex
- Why? Get student to recognize hex is easier
- Extra: (older students)
- "hexadecimal" is a mix of greek/latin roots
- It should be "sexadecimal"
- Compare sexagesimal for base-60
- Can you guess why hex was chosen?


## Boolean Logic

(4th-5th grade)

## Boolean logic

- Given a set of true/false statements:
- isRaining - true if it's raining outside
- hasRaincoat - true if you have a raincoat
- Formal way of combining statements:
- getsWet = isRaining AND NOT hasRaincoat
- Basis of logical "thought"
- Two values : true/false. Sounds like binary.
- 0 = false; 1 = true


## Boolean logic activity

- Create logical statements about objects/people in the classroom
- isRed, isFlat, isBiggerThanMyHead, ...
- isBoy, hasGreenShirt
- Apply the statements to other objects and state whether they are true or false


## Boolean operations

- Four basic operations that can be applied to statements:
- NOT
- AND
- OR
- XOR


## NOT

- Changes true to false and vice versa
- Given
- isCat $=$ true
- Then
- NOT isCat = false
- In English, we say "is not a cat"
- In logic, we say "not is-a-cat"


## AND

- a AND b is true only if $\mathbf{a}$ and $\mathbf{b}$ are both true
- Given statements:
- isCat
- hasStripes
- Then
- isCat AND hasStripes
- is true for striped cats
- is false for spotted cats or striped dogs


## OR

- $\mathbf{a}$ OR $\mathbf{b}$ is true if either $\mathbf{a}$ or $\mathbf{b}$ (or both) are true
- Given statements:
- isCat
- hasStripes
- Then
- isCat OR hasStripes
- is true for striped cats, spotted cats, striped dogs
- is false for spotted dogs


## XOR

- $\mathbf{a} X O R \mathbf{b}$ is true if either $\mathbf{a}$ or $\mathbf{b}$ (but not both) are true
- Given statements:
- isCat
- hasStripes
- Then
- isCat XOR hasStripes
- is true for spotted cats, striped dogs
- is false for striped cats, spotted dogs


## Boolean logic activity

- Activity \#1:
- Create more logical statements using NOT, OR, AND, XOR
- Activity \#2:
- Take a collection of objects:
- Items in room, cards from "Guess Who?", ...
- Choose a few of the objects
- Create a statement that will be true only for those objects and false for all others


## Boolean logic example

- Cockatrice in the game Nethack
- Petrifies you (turns you to stone) if you touch it
- Logic from game:
petrify = (handAttack AND NOT wearGloves)
OR (kick AND NOT wearBoots
OR (headbutt AND NOT wearHelmet)
OR (hug AND NOT (wearGloves AND wearCloak))
OR bite OR sting OR suckBrain OR swallow
- Note that in the game you can polymorph into monsters (like mind-flayer) that have special attacks (like brain-suck)


## Boolean logic activity

- Design a simple game that contains a special object.
- Create logical expressions that define all the interactions with that object
- pickUp, drop (on ground, in water), eat, hit, ...
- Trade with a student and see if they can come up with conditions you didn't think of
- Do the same thing for an object in a video game that you've played


## Boolean logic - final notes

- Two final notes on boolean logic:
- Truth tables
- Used to summaries logic statements
- Useful when solving logic problems
- Logic gates
- Graphical representation of logical operations


## Truth tables

- Way of presenting logical statements
- Enumerating all possible outcomes

| isCat | hasStripes | IsCat <br> AND <br> hasStripes | IsCat <br> OR <br> hasStripes | IsCat <br> XOR |
| :---: | :---: | :---: | :---: | :---: |
| hasStripes |  |  |  |  |$|$

## Logic gates

NOT


| $A$ | $Q$ |
| :--- | :--- |
| 0 | 1 |
| 1 | 0 |

AND


| $A$ | $B$ | $Q$ |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |


| $A$ | $B$ | $Q$ |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |


| $A$ | $B$ | $Q$ |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

XOR


## Logic gate activity

- Draw one of your logical statements from before as a connected series of gates:


Transistors
(4th-5th grade)

## Activity

- Good time to cover/review basic electricity:
- Light bulbs, switches, batteries
- Electrical current, electrons
- Materials: conductors vs. insulators
- No need to cover resistors or anything more complex


## Binary values in electricity

- Electronic devices typically have 2 electrical states:
- Power (red wire)
- Ground (black wire)
- For a battery: (-) Ground (+) Power
- Two states. Sounds like binary:
- 0 = Ground; 1 = Power


## Transistors

- What are transistors?
- Electrical switches
- Similar to the light switches:
- Two positions: on / off
- On: electricity flows to lightbulb
- Off: electricity does not flow to lightbulb
- Difference:
- Switch is controlled electrically
- So a switch can control another switch


## Transistor



If switch is ON
$A$ and $B$ are connected

If switch is OFF
$A$ and $B$ are not connected

## Types of transistors

- Lots of different types:
- FET : field effect transistor
- BJT : bipolar junction transistor
- But we don't care:
- Just pick one that is easy to explain:
- MOSFETs : metal oxide semiconductor FET
- Also one of the more common types


## Semiconductors

- Semiconductors:
- Not good conductors
- Not good insulators
- What good are they?
- With the right tricks, they can switch from one state to the other
- This is how transistors work


## CMOS

- CMOS (complementary MOS):
- Uses nMOS and pMOS transistors
- Arranged in a complementary fashion



## Building logic gates from transistors

- Invertor (boolean NOT) is the simplest
- Requires 2 transistors: 1 nMOS \& 1 pMOS
- Other gates can be built with 2 of each type:
- NAND = NOT AND
- NOR = NOT OR
- We can combine gates:
- AND = NOT NAND
- OR = NOT NOR


## CMOS inverter




## Transistor activity - inverter

- Use cards to build logic gates from transistors
- Red/black markers for power/ground
- Students trace flow of " 0 "s and " 1 "s


How Computers Add (5th-6th grade)

## How computers add

- Computers add the same way humans add
- Except in binary


## How do you teach addition?



Memorize this table!

## Teaching addition (review)

- First teach adding 2 single-digit numbers
$-1+1=2,3+5=8, \ldots$
- Later, multi-digit answers:

$$
\text { - } 7+8=15
$$

- Then expand to adding multi-digit numbers:
$-18+23=41$
- With carry


## Single digit addition

## 4



Add the 2 blue numbers
Produce the single-digit sum (green) and the carry (yellow)
carry sum

## Sum table \& carry table

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 |
| A | 5 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 |
|  | 4 |  |  |  |  |  |  |  |  |  |
| 6 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 |
| 7 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 8 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 9 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Sum table

|  | B |  |  |  | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 7 | 8 | 9 |  |  |  |  |  |  |  |
| A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## Binary addition

- Same general idea as with decimal
- Uses binary addition table
- Just like the decimal addition table


## Binary addition table

That's all

## Binary sum and carry tables

- Binary addition table:

- As before, break into sum and carry tables:


Carry


Sum

## More logically...

Re-writing these 2 tables more "logically"

|  |  | $B$ |  |
| ---: | ---: | :--- | :--- |
| Carry | + | 0 | 1 |
| $A$ | 0 | 0 | 0 |
|  | 1 | 0 | 1 |


|  | $B$ |  |  |
| :---: | :---: | :---: | :---: |
| Sum $^{\prime}$ | + | 0 | 1 |
| $A$ | 0 | 0 | 1 |
|  | 1 | 1 | 0 |

We get:

| A | B | Carry | Sum |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

## Adding with logic gates

Compare this with our boolean logic gates:

| A | B | Carry | Sum | AND | XOR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 |

Carry is AND
Sum is XOR

## The half-adder



## Half-adder activity

- Use cards like with the transistor
- AND, XOR gate cards + yarn for "wire"
- Build a half-adder
- Verify that it produces the binary addition table


## Adding multiple digits

- A half-adder only adds 2 single digit numbers
- It only does "half" the job
- We need to be able to handle both carries
- Carry coming in from previous digit
- Carry going out to next digit


## Multi-digit addition (decimal)



$$
294
$$

## Supporting carries with a full-adder

- A full-adder extends the half-adder by adding support for the carry
- Two steps:
- Add the two numbers (as before)
- Add the carry to the result
- Built from 2 half-adders
- Plus an OR gate to combine the carries


## Full-adder

## Carry (in)



## Full-adder activity

- Using cards, construct a full-adder and verify it works
- Investigate why we can OR the carries together
- Each half-adder produces a carry
- The full-adder just ORs the 2 carries together
- Is it possible for both carries to be set at the same time?


## Ripple carry adder

- Full adders connected together
- Propagate the carry from one digit to the next
- Just like we teach students to do for decimal
- Any number of full adders can be connected
- 4 full adders supports adding two 4-digit numbers


## Ripple carry adder



## Ripple carry adder activity

- With N students, give each one a full-adder to construct.
- Connect them end-to-end to build a ripple carry adder
- Add 2 N-digit binary numbers


## Resources

- Slides will be posted at:
- cse4k12.blogspot.com
- Updates will also be posted there
- We are in the process of converting these activities into worksheets/activity sheets.
- Email contact:
- garykac @ either "gmail" or "google" .com

